MILP APPROACH FOR THE DESIGN OF VERTICAL VAPOR-LIQUID SEPARATION VESSELS - COMPARISON WITH HEURISTICS

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***Abstract*— In this article we compare results from different heuristics approaches for the design of VLE separation vessels. In addition, we present an MILP approach that embeds the aforementioned heuristics and considers the discrete nature of the geometric variables. We show that different heuristics render different results and, while results from heuristics and MILP often coincide, significant departures occur.**

***Keywords*— Phase Separator Design, Optimization.**

I. INTRODUCTION

Vapor-liquid phase separators are widely used in oil refineries, natural gas processing plants, petrochemical and chemical plants, etc. (Meyers, 1997; Kayode Coker, 2010; Speight, 2011). In classical textbooks (Silla, 2003; Couper *et al*., 2005; Stewart and Arnold, 2008; Datta, 2008; Towler and Sinnott, 2008), heuristic procedures are proposed. Another approach for the design uses computational fluid dynamic (CFD) (Misra *et al*., 2017; Ghaffarkhah *et al*., 2017). We focus our analysis on the former approaches, as usually employed in practice and we reformulate it in a form of an optimization procedure.

In this article, results using different heuristics for the design of vapor-liquid separation vessels are compared and an alternative optimization procedure is presented. Design principles used by traditional heuristics are presented first. Then, the traditional heuristic procedures and the MILP optimization procedure is presented. Solutions of the MILP procedure are compared with the heuristics' solutions highlighting important discrepancies in some cases, which indicate that the utilization of mathematical programing can attain solutions with lower costs.

II. DESIGN PRINCIPLES

In this section, we list all the constraints used to design vertical VLE phase separators (Figure 1). The design variables of a vertical separator are: diameter, height, type headers and wall thickness. For comparison, we pick the following heuristic procedures: Silla (2003), Couper *et al*. (2005), who follows Evans (1980), and Towler and Sinnott (2008). These design procedures are focused on the determination of the vessel diameter based on the separation of the droplets from the vapor flow.

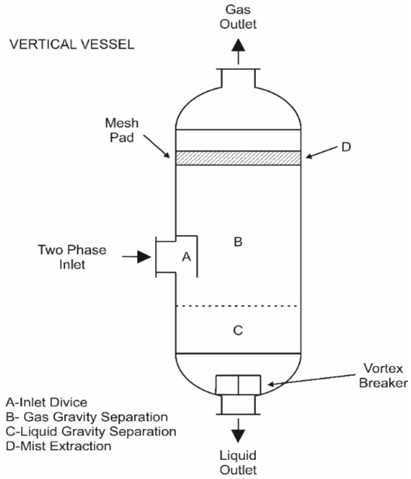

Figure 1. Vertical Phase Separator.

A. Vessel Diameter

It is obtained based on the vapor velocity v_v

$$D_{vessel} = \left(\frac{4\hat{Q}_v}{\pi v_v}\right)^{1/2} \quad (1)$$

where $\hat{Q}_v$ is the operating vapor volumetric flow rate (parameters are represented with a "^" on the top).

When the diameter is small such that regular scheduled pipes can be used, rounding off the diameter to the next available commercial value is done. When plates are used, there also exist recommendations to round up to the next standard value (Silla, 2003; Couper *et al*., 2005).

B. Terminal Velocity

All three sources recommend determining the terminal velocity (or settling velocity) of a droplet of a certain size ($\hat{d}_{d,crit}$) first. Liquid droplets of smaller size than the cut size remain part of the vapor, eventually coalescing in the mist extractor when present. Balancing the buoyancy, drag forces and gravitational forces, one obtains (Silla, 2003; Stewart and Arnold, 2008; Datta, 2008):

$$v_v = \left(\frac{4\,\hat{d}_{d,crit}\,\hat{g}}{3\,C_D}\right)^{1/2}\left(\frac{\hat{\rho}_l-\hat{\rho}_v}{\hat{\rho}_v}\right)^{1/2} \quad (2)$$

where C_D is the drag coefficient. Typically, the above formula is rewritten introducing a coefficient K_v:

$$v_v = K_v\left(\frac{\hat{\rho}_l-\hat{\rho}_v}{\hat{\rho}_v}\right)^{1/2} \quad (3)$$

In this article, we just use values of K_v, and we do not pick values of $\hat{d}_{d,crit}$ or C_D (left for future work). In addition, we do not consider the use of demisters.

Many authors suggest different fixed approximate values of K_v. Gerunda (1981) says (without citing any source), that for most systems, the value of K_v varies between 0.1 and 0.35 ft/s (0.03045 and 0.107 m/s), the value of $\widehat{K}_v$ = 0.03405 ft/s (0.01038 m/s) being the most adequate for designs without demisters. In turn, Towler and Sinnott (2008) suggest using $\widehat{K}_v$ = 0.0105 m/s for vessels without demister. Finally, Silla (2003) takes the extreme values of the range used by Gerunda (1981), that is, $\widehat{K}_v$ = 0.1 ft/s (0.03045 m/s) for separator without demister. Other authors attempt to calculate it directly through proposed formulas. In particular, Watkins (1967) proposed, not citing any source, that $\widehat{K}_v$ be a function of $\widehat{F}_{lv} = (\widehat{W}_l/\widehat{W}_v)\sqrt{\hat{\rho}_v/\hat{\rho}_l}$ ($\widehat{W}_v$ and $\widehat{W}_l$ are the vapor and liquid mass flowrates, respectively). Watkins (1967) claims that his curve corresponds to 5% liquid entrainment with no demister and 85% flooding. Blackwell (1984) proposed an analytical expression (a 5th degree polynomial based on (ln $\widehat{F}_{lv}$), which we use here.

C. Vapor Height

Vapor height, the length between the liquid surface to the top, is obtained using a set of heuristics that vary from author to author, all presumably attempting to allow some length to let parallel flow to develop. Watkins (1967) proposed 36”+0.5 of nozzle diameter above the inlet nozzle center and 12”+0.5 nozzle diameter below the inlet nozzle center. When there is no demister, Couper *et al.* (2005) show recommendations in a figure: a minimum of 48” above the inlet nozzle center, and a minimum of 18” between it and the liquid level increasing, values far larger than the ones proposed by Watkins (1967). In addition, in the text, they do not refer to this figure, but rather to another figure presented for the case of the use of demisters. Finally, according to Couper *et al.* (2005) and Watkins (1967), the aforemen-tioned nozzle diameter should be selected such as the nozzle two-phase velocity ($\hat{u}_n$) is between minimum and maximum values, calculated to be 60 and 100 times the value of $\sqrt{\hat{\rho}_{mix}}$. We use a conservative velocity 10% below the maximum, that is:

$$\hat{u}_n(\text{ft/s}) = 90\sqrt{\hat{\rho}_{mix}} \quad (4)$$

In our comparisons, for the case of using the heuristics by Cooper *et al.* (2005), the recommendations are a minimum of 48” above the inlet nozzle center, and a minimum of 18” between it and the liquid level (the minimum values proposed by Watkins (1967)). Thus, we have $H_v = 1.6764$ m (66").

In turn, Towler and Sinnott (2008) only depict the case when a demister is installed. The height is obtained adding 0.4 m (minimum) for allowing demister installation and the space above, one vessel diameter from the center of the nozzle to the demister (with a minimum of 1 m), and half diameter from the center of the nozzle to the liquid level. (0.6 m minimum). To this, they propose to add space for the inlet fittings (H_{inlet}), and above that, certain room for the vapor flow to develop (H_v).

Finally, the heuristic proposed by Silla (2003) is based on the sum of $H_{inlet-liq}$ and $H_{disengage}$, the same way as in the case of Towler and Sinnott (2008):

$$H_v = H_{inlet-liq} + H_{disengage} \quad (5)$$

Towler and Sinnott (2008) suggest

$$H_{inlet-liq} = \max\{0.5\,D_{vessel}, 0.6\text{ m}\} \quad (6)$$

$$H_{disengage} = \max\{D_{vessel}, 1\} \quad (7)$$

In turn, Silla (2003) suggests

$$H_{inlet-liq} = \max\{0.5\,D_{vessel}, 0.6096\} \quad (8)$$

$$H_{disengage} = \max\{D_{vessel}, 0.9144\} \quad (9)$$

All the aforementioned heuristics are based on the inability of predicting in detail when a velocity field is uniform and upwards. This matter is left for future work.

D. Liquid Level

It is obtained using a given residence time $\hat{t}_s$. The values vary from 5 to 20 min (Cooper *et al.*, 2005), who take that recommendation from Walas (1988), 10 min for Towler and Sinnott (2008) and 5 min for Silla (2003). All values are subject to a minimum value (0.61 m), as recommended by Silla (2003) or 0.3 m as suggested by Towler and Sinnott (2008) to accommodate a level measurement device. In turn, Couper *et al.* (2005) do not mention a minimum specifically, only citing vaguely the need for liquid hold-up. Thus, one can safely assume that a level measurement device will be installed. Therefore,

$$H_l = \max\left\{\frac{4\hat{Q}_l\hat{t}_s}{\pi D_{vessel}^2}, \widehat{H}_{l,min}\right\} \quad (10)$$

E. Slenderness

The H_{vessel}/D_{vessel} restrictions can be traced back to Watkins (1967), also cited by many as the earliest source. Watkins’ article states that the limits are based on the fact that “as diameter decreases, the shell thickness decreases and vessel length increases.” Then he adds: “At some point between H_{vessel}/D_{vessel} ratios of 3 and 5, a minimum weight will occur, and this will result in minimum cost.” Watkins (1967) also mentions that when H_{vessel}/D_{vessel} is larger than 5, it is more convenient to use a horizontal separator, based on the notion that horizontal separators can better hold liquid.

Couper *et al*. (2005) and Silla (2003) state that H_{vessel}/D_{vessel} needs to be higher than 3 and lower than 5, as first suggested by Watkins (1967). We will assume the same for Towler and Sinnott (2008), although it is not explicitly mentioned.

Thus, if H_{vessel}/D_{vessel} > 5, then the heuristics suggest that the diameter be increased a certain percentage (defined by the experienced designer) until it complies. With this new vessel diameter, the vessel height is recalculated. The procedure is repeated, until finding values of diameter and height that satisfy the previous relationship. Conversely, if H_{vessel}/D_{vessel}<3, then the height is increased keeping the diameter constant (since it is in the minimum value), until the ratio is larger or equal to 3.

F. Wall Thickness of Shell and Heads

It depends on the fabrication method, for which the most popular choices are a portion of a large pipe or rolled steel plates, later welded. For the case of the use of pipes, the diameter and wall thickness come in discrete choices, related to the standardized pipe schedule also taking into account pressure. In the case of rolled steel plates, pressure also dictates the thickness to be used, which also comes in standardized discrete choices.

The top and bottom heads choices are elliptical or hemispherical, with also standardized discrete thickness choices. The following formulas are used (Silla, 2003; Couper *et al.*, 2005; Stewart and Arnold, 2008):

$$t_{shell} = \max\left\{\frac{\hat{P}\frac{D_{vessel}}{2}}{\hat{S}\hat{E}-0.6\hat{P}}, \hat{t}_{min}\right\} \quad (11)$$

$$t_{h-ellipsoidal} = \max\left\{\frac{\hat{P}D_{vessel}}{2\hat{S}\hat{E}-0.2\hat{P}}, \hat{t}_{min}\right\} \quad (12)$$

$$t_{h-hemispherical} = \max\left\{\frac{\hat{P}\cdot\frac{D_{vessel}}{2}}{2\hat{S}\hat{E}-0.2\hat{P}}, \hat{t}_{min}\right\} \quad (13)$$

which means that a choice between of head needs to be made. The value of $\hat{t}_{min}$ is usually 3/32" (2.38 mm) or higher if an allowance for corrosion is added: Stewart and Arnold (2008) recommend adding ¼ in. On the other hand, Silla (2003) considers this value excessive. The thickness for hemispherical heads is lower than the one for ellipsoidal heads for the same diameter and pressure ($\hat{P}$) and other fixed parameters like maximum allowable stress values ($\hat{S}$) and welding efficiency ($\hat{E} = 1$ for double welding with X-ray and 0.6 for simple welding with no X-ray). In the case of vessels made from pipes, which do not exhibit a welding joint, they are still welded to the heads, or to the inlet nozzle. Note that hemispherical head has more volume than the ellipsoidal head for the same diameter and the same thickness. In turn, the ellipsoidal head renders twice the thickness for the same pressure and diameter. An additional heuristics states that above 150 bar it is advisable to use spherical heads (Silla, 2003).

G. Vessel Volume

The volume of shell material is determined as the shell volume plus heads volumes, for ellipsoidal and hemispherical head, respectively:

$$V_{shell} = \pi(D_{vessel}t_{shell} + t_{shell}{}^2)H_{vessel} \quad (14)$$

$$V_{head} = \frac{\pi}{12}\left(6D_{vessel}{}^2t_h + 12D_{vessel}t_h{}^2 + 8t_h{}^3\right) \quad (15)$$

$$V_{head} = \frac{\pi}{6}\left(6D_{vessel}{}^2t_h + 12D_{vessel}t_h{}^2 + 8t_h{}^3\right) \quad (16)$$

H. Cost

The cost is composed as the cost of metal. To compare we use the total mass of material employed to build the vessel.

III. RESULTS OBTAINED USING HEURISTICS

A. Straight Comparisons

We compare traditional heuristic design procedures to discuss the impact of the different choices for fixed $\hat{K}_v$ values and the different restrictions associated to the height. The discrete values of diameter and thickness for pipes we used correspond to STD, XS and XXS and we do not include schedules 10, 40 and 60. For diameter

Table 1: Data for Example 1.

Parameter	Value	Parameter	Value
$\hat{Q}_v(\mathrm{m^3/s})$	1.4157	$\hat{\mu}_v(\mathrm{Pa\cdot s})$	$1.8\cdot10^{-5}$
$\hat{Q}_l(\mathrm{m^3/s})$	0.0117987	$\hat{t}_s(\mathrm{s})$	600
$\hat{\rho}_l(\mathrm{kg/m^3})$	999.552	$\hat{P}(\mathrm{psig})$	30
$\hat{\rho}_v(\mathrm{kg}/m^3)$	1.201385	$\hat{S}(\mathrm{psi})$	17500

Table 2: Heuristics results - Example 1 (No demister)

Model / Variables	From Couper *et al.* (2005)	Couper *et al.* (2005)	Towler and Sinnott (2008)	Silla (2003)
D_{vessel} (m)	1.3717	1.5049	1.5049	1.5049
H_{vessel} (m)	6.4617	5.6564	6.24	6.24
t_{shell}, t_h (mm)	9.53	9.53	9.53	9.53
$\hat{K}_v$ (m/s)	0.04267	0.1058	0.1058	0.1058
H_{vessel}/D_{vessel}	4.71	3.76	4.14	4.14
Mass (kg)	2562.22	2568.55	2776.65	2776.65

higher than 60", we used rolled steel plates (of standard thickness) discretizing the external diameter using 2 inches steps. We use $\hat{\rho}_{steel}$ =7900 kg/m^3 for ASTM SA516 G70.

The first example is taken from Couper *et al*. (2005) where liquid water is separated from air at atmospheric conditions (Table 1).

Results are shown in Table 2, including the original results (D_{vessel} and H_{vessel}) obtained by Couper *et al.* (2005), where we assume ellipsoidal head and a thickness of 0.00953 m. We note that, originally, Couper *et al*. (2005) (first column), used a value of $\hat{K}_v$, different than the one we obtain from the Blackwell formula and that all results include ellipsoidal heads.

We also made the adjustments needed to abide by the slenderness constraints (H_{vessel}/D_{vessel} ratio), as the heuristics suggest.

We illustrate the adjustments made as follows: using the Couper et al's heuristics one first obtains D_{vessel} = 0.7876 m, H_{vessel}= 16.2070 m, and H_{vessel}/D_{vessel}= 20.5777. Since the ratio obtained H_{vessel}/D_{vessel} is higher than the maximum ratio recommended according to the heuristics, and the heuristic procedure calls for changing the diameter in this situation, we try that until $H_{vessel}/D_{vessel} \leq 5$. The sequence of diameters and heights tried is the following: (D_{vessel}, H_{vessel}), = (0.8386 m, 14.4934 m), (0.8448 m, 14.3055 m), … (0.8949 m, 12.9314 m),… (1.4986 m, 5.6899 m), with this last complying with the slenderness constraint.

The corresponding thickness for D_{vessel}=1.4986 m (60-XS) is 12.7 mm , with a corresponding mass of 3430.36 kg. However, if one picks the next diameter (60-STD) D_{vessel}=1.5049 m, one gets a thickness of 9.53 mm, which leads to a smaller weight (2568.55 kg), because STD pipes have smaller thickness. This is not mentioned in the heuristics we explored, and it makes a significant difference (28.04 % lower mass).

The second example is taken from Towler and Sinnott (2008). Data for this example (water/steam) are shown in Table 3. For the heuristics, we used $\hat{K}_v$ = 0.12602 m/s

Table 3: Data for Example 2.

Parameter	Value	Parameter	Value
$\hat{Q}_v(\mathrm{m}^3/\mathrm{s})$	0.257	$\hat{\mu}_v(\mathrm{Pa}\cdot\mathrm{s})$	$1.4521\cdot 10^{-5}$
$\hat{Q}_l(\mathrm{m}^3/\mathrm{s})$	0.0003	$\hat{t}_s(\mathrm{s})$	750
$\hat{\rho}_l(\mathrm{kg/m}^3)$	926.4	$\hat{P}(\mathrm{psig})$	68.32
$\hat{\rho}_v(\mathrm{kg/m}^3)$	2.16	$\hat{S}(\mathrm{psi})$	17500

Table 4: Heuristics results - Example 2.

Model / Variables	From Towler and Sinnott (2008)	Couper *et al.* (2005)	Towler and Sinnott (2008)	Silla (2003)
D_{vessel} (m)	1.25	0.5399	0.5399	0.5399
H_{vessel} (m)	3.75	2.4626	2.3862	2.3102
t_{shell}, t_h (mm)	9.53	9.53	9.53	9.53
$\widehat{K}_v$ (m/s)	0.0105	0.12602	0.12602	0.12602
H_{vessel}/D_{vessel}	3	4.5613	4.4198	4.2790
Mass (kg)	1492.38	391.43	381.50	371.63

(determined using the Blackwell equation). Adjustments to satisfy the H_{vessel}/D_{vessel} ratio restriction are only necessary for the heuristics of Towler and Sinnott (2008). The results are in Table 4, where the original results from Towler and Sinnott (2008) were completed using their heuristics (first column). Heads are all ellipsoidal.

In reporting the original results from Towler and Sinnott (2008) in the first column, we note that Towler and Sinnot added 0.4 m to the height. When we used the heuristics (third column), this distance is not added, because there is no demister here. If this value is not considered in the first column, then the height and vessel mass would be H_{vessel} = 3.35 m and *Mass* =1403.00 kg.

B. Danger of using arbitrarily selected values of $\widehat{K}_v$

Example 1 was taken from Couper *et al.* (2005), where they used a calculated value of $\widehat{K}_v$ = 0.04267 m/s (0.14 ft/s), corresponding to $\hat{v}_v$ = 1.2301 m/s, which is about 40 % of the $\hat{v}_v$ obtained using the Blackwell equation. Thus, D_{vessel} = 1.5049 m, H_{vessel}= 5.6564 m and vessel mass of 2568.55 kg, the same as in the second column of Table 2.

For this example 2, Towler and Sinnot (2008), used a fixed value of $\widehat{K}_v$= 0.0105 m/s, which corresponds to $\hat{v}_v$ = 0.2147 m/s, contrasting with 2.607 m/s obtained with $\widehat{K}_v$ using the Blackwell equation. We obtained D_{vessel} = 1.5049 m, and adjusting the vessel height for slenderness, we get H_{vessel} = 4.5147 m and a weight of 2159.61 kg.

Both examples illustrate the dangers of using arbitrarily selected values of $\widehat{K}_v$. The important differences of mass obtained for Towler and Sinnott (2008) heuristics, using fixed ad-hoc values of $\widehat{K}_v$ and values obtained using the Blackwell equation (2159.61 kg and 381.50 kg, respectively) highlight the danger of using the formula.

We conclude this section stating that the different heuristics offer intricacies that are hard to unify in a single universal heuristics-based recipe. In addition, these heuristics are misleading when adjustments for slenderness are made. Moreover, even when adjustments for slenderness are not needed, the results can be substantially different because one can still pick larger diameters that lead to smaller mass. Thus, for these two reasons, we recommend NOT using heuristics.

IV. OPTIMIZATION PROCEDURE

We now present a mixed-integer linear programming (MILP) formulation for solving the design problem through mathematical programming. The model takes into account the above presented heuristics regarding heights, but it does not force the procedural suggestions, i.e. the proposed formulation can identify the optimal solution according to the literature equations employed in the practical design. We introduce binary variables z_{he} and z_{hh} to indicate the use of ellipsoidal or hemispherical heads.

$$\min Cost = (V_{shell} + 2\, V_{head})\hat{\rho}_{steel}\hat{C}_{steel} \quad (17)$$

s.t.

$$D_{vessel} \geq \left(\frac{4\hat{Q}_v}{\pi\hat{v}_v}\right)^{1/2} \quad (18)$$

$$D_{vessel} \geq \widehat{D}_{min} \quad (19)$$

$$H_l \geq \frac{4\hat{Q}_l\hat{t}_s}{\pi D_{vessel}^2} \quad (20)$$

$$H_l \geq \widehat{H}_{l\_min} \quad (21)$$

$$H_{vessel} \geq H_l + H_v \quad (22)$$

$$H_v = \begin{cases} 1.6764 \text{ (Couper } et\ al.\text{)} \\ (H_{inlet-liq} + H_{disengage}) \text{(Towler, Sinnott)} \\ (H_{inlet-liq} + H_{disengage}) \text{ (Silla)} \end{cases} \quad (23)$$

$$H_{inlet-liq} \geq \begin{cases} \max\{0.5D_{vessel}, 0.6\} \text{ (Towler; Sinnott)} \\ \max\{0.5D_{vessel}, 0.6096\} \text{ (Silla)} \end{cases} \quad (24)$$

$$H_{disengage} \geq \begin{cases} \max\{D_{vessel}, 1\} \text{ (Towler; Sinnott)} \\ \max\{D_{vessel}, 0.9144\} \text{ (Silla)} \end{cases} \quad (25)$$

$$3 \leq H_{vessel}/D_{vessel} \leq 5 \quad (26)$$

$$t_{shell} \geq \frac{\hat{P}\frac{D_{vessel}}{2}}{\hat{S}\hat{E}-0.6\hat{P}} \quad (27)$$

$$t_{shell} \geq \hat{t}_{min} \quad (28)$$

$$t_h \geq \frac{\hat{P}\cdot D_{vessel}}{2\cdot\hat{S}\cdot\hat{E}-0.2\cdot\hat{P}} z_{he} + \frac{\hat{P}\cdot\frac{D_{vessel}}{2}}{2\cdot\hat{S}\cdot\hat{E}-0.2\cdot\hat{P}} z_{hh} \quad (29)$$

$$t_h \geq \hat{t}_{min} \quad (30)$$

$$V_{shell} = \pi(D_{vessel}t_{shell} + t_{shell}^2)H_{vessel} \quad (31)$$

$$V_{head} = \frac{\pi}{12}\left(6D_{vessel}^2 t_h + 12D_{vessel}t_h^2 + 8t_h^3\right)z_{he} + \frac{\pi}{6}\left(6D_{vessel}^2 t_h + 12\, D_{vessel}t_h^2 + 8t_h^3\right)z_{hh} \quad (32)$$

$$z_{he} + z_{hh} = 1 \quad (33)$$

Many geometric variables (x) have several discrete options $\widehat{xd}_i$ according to standard/commercial alternatives (e.g. diameters, wall thicknesses, lengths, etc). Thus, we use binary variables y_i, and write x as follows:

$$x = \textstyle\sum_i \widehat{xd}_i\, y_i \quad (34)$$

$$\textstyle\sum_i y_i = 1 \quad (35)$$

After the substitution of the discrete variables by its binary representation in the mathematical expressions of the vessel model, we get terms of the form $p^{n1}q^{n2}\cdots z^{nm}$ that are substituted as follows:

$$p^{n1}q^{n2}\cdots z^{nm} = [\textstyle\sum_i \widehat{pd}_i\, yp_i]^{n1}\,[\sum_j \widehat{qd}_j\, yq_j]^{n2}\,[\sum_k \widehat{zd}_k\, yz_k]^{nm} \quad (36)$$

Because Eqs. (34-35) render only one binary variable equal to 1, one can write:

Table 5: MILP results - Example 1.

Model / Variables	Couper *et al.* (2005)	Towler and Sinnott (2008)	Silla (2003)
D_{vessel} (m)	1.5685	1.5685	1.5685
H_{vessel} (m)	5.34	6.02	6.02
t_{shell}, t_h (mm)	3.2	3.2	3.2
$\widehat{K}_v$ (m/s)	0.1058	0.1058	0.1058
H_{vessel}/D_{vessel}	3.41	3.84	3.84
Mass (kg)	857.36	941.25	941.25

$$p^{n1} q^{n2} \cdots z^{nm} = \sum_{i,j,..k} \widehat{pd}_i^{\,n1} \widehat{qd}_j^{\,n2} \ldots\ldots \widehat{qd}_k^{\,nm}\, yp_i\, yq_j \ldots yz_k \quad (37)$$

Therefore, the reformulated model is now composed by several expressions containing multiple summations of products of binary variables and a few continuous variables. Finally, the products of binary variables are linearized using standard procedure: First Eq. (37) is rewritten as follows

$$p^{n1} q^{n2} \cdots z^{nm} = \sum_{i,j,..k} \widehat{pd}_i^{\,n1} \widehat{qd}_j^{\,n2} \ldots \widehat{qd}_k^{\,nm}\, wp_{i,j,\ldots,k} \quad (38)$$

and the following equations are added:

$$wp_{i,j,\ldots,k} \leq yp_i \quad (39)$$

$$wp_{i,j,\ldots,k} \leq yq_j \quad (40)$$

$$\ldots\ldots\ldots$$

$$wp_{i,j,\ldots,k} \leq yz_k \quad (41)$$

$$wp_{i,j,\ldots,k} \geq yp_i + yq_j + \ldots + yz_k - (m-1) \quad (42)$$

where m is the number of binary variables participating in the product.

The variables in our model are discretized as follows:

$$D_{vessel} = \sum_{std}^{stdmax} \widehat{Vessel}_{std,intd}\, yvessel_{std} \quad (43)$$

$$t_{shell} = \sum_{std}^{stdmax} \widehat{Vessel}_{std,tshell}\, yvessel_{std} \quad (44)$$

$$t_h = \sum_{std}^{stdmax} \widehat{Vessel}_{std,th}\, yvessel_{std} \quad (45)$$

$$\sum_{std}^{stdmax} yvessel_{std} = 1 \quad (46)$$

All MILP procedures usually reproduce the heuristics, with the exception of certain cases that we explore next.

V. RESULTS OF MILP PROCEDURES

The MILP procedure was run using different models. The results of Example 2 are the same as those found by the heuristics (Table 4), while the results for Example 1 are shown in Table 5. All heads are ellipsoidal.

The results show that for the Example 1, the MILP approach reaches better results that the solution found by the heuristics. We discuss this in the next section.

VI. DEPARTURES FROM HEURISTICS

There are two departures of interest. First, those that derive from continued testing of larger diameter after the tests and eventual adjustments because of slenderness constraints are performed and second, discrepancies related to different liquid residence times.

A. Diameter larger than heuristics results

The final diameter departure from the calculated minimum diameter is usually small. If slenderness constraints are enforced, this diameter sometimes changes. However, even if the latter case occurs, if one can continue increasing diameter and obtain a smaller weight.

In Example 1, if one ignores stopping as soon as slenderness is within limits, and continues looking for larger diameters, which in this case are plates, then one obtains D_{vessel}= 1.5685 m, H_{vessel} = 5.34 m (a smaller height), with a mass of 857.36 kg. vs. 2568.55 kg, both results abiding by slenderness constraints. This is actually picked up by the MILP.

B. Diameter larger because of Liquid Surge Time

There is, however another case where departures from heuristics take place and it is when the liquid storage capacity renders larger diameters, a situation that is captured by the MILP, but not considered in step-by-step heuristics procedures. For example, for Couper *et al.* (2005) model, when one uses $\hat{t}_s = 20$ min instead of $\hat{t}_s$ =10 min, both examples exhibit diameters that depart significantly from the minimum diameter. This is shown in Table 6, and in Fig. 2. Indeed, according to heuristics, one should stay close to the minimum diameter obtained using Eq. (1) by simply adjusting to the next standard diameter. The MILP breaks with this heuristic selecting a much larger diameter. In other words, when the diameter increases, the liquid storage capacity increases and the height decreases (because liquid height decreases), reaching a point where the vessel weight is the lowest, before it starts increasing.

Table 6: Minimum vessel diameter - Example 1 and 2.

	Example 1	Example 2
Diameter Eq.(1)	0.77 m	0.35 m
Optimum Diameter MILP procedure	1.823 m	0.6409 m
Optimum Height MILP procedure	7.1038 m	2.7918 m

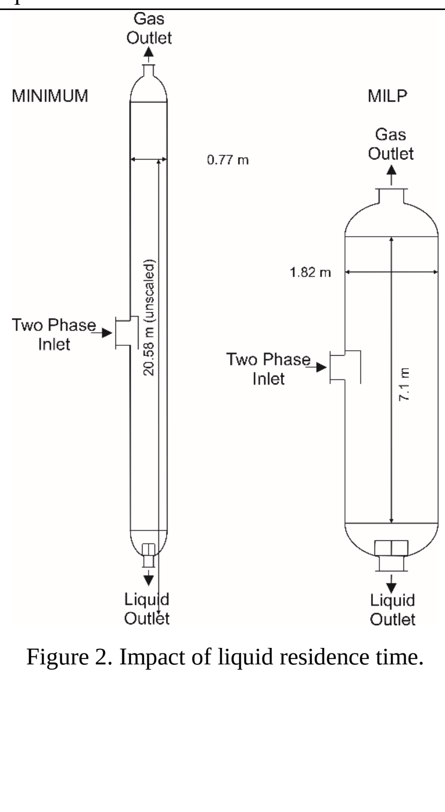

Figure 2. Impact of liquid residence time.

VII. EXHAUSTIVE ENUMERATION

The above MILP procedure is very useful, especially if it is embedded in larger models where more equipment is optimized simultaneously. For stand-alone calculations, it can be applied by an enumeration procedure, where each combination of diameter, thickness and head option is considered.

VIII. CONCLUSIONS

The use of heuristics for the design of vertical vapor liquid separators has been revisited finding that different authors provide different recommendations, rendering different designs. We also developed an MILP procedure that can replace the use of heuristics. We found that the MILP procedure can reach better results in some cases. Therefore, the proposed approach becomes an automatic procedure, which without the need of direct human intervention, can attain capital cost reductions for vessel design, thus improving the power of heuristic-based design procedures. In addition, we also explored the impact of some design parameters.

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